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I. **Solution by A. H. HOLMES, Professor of Mathematics, Bowdoin College, Brunswick, Me.**

Let ABC be the triangle, O the center of the inscribed circle, and OD the perpendicular from O to the side AB, its radius; let $\angle BOD=\theta$ and $\angle AOD=\phi$. Then we have $\tan\theta+\tan\phi=\frac{54}{7}$. But $\phi+2\theta=180°$.

$$\therefore\ \tan\phi=-\tan2\theta=-\frac{2\tan\theta}{1-\tan^2\theta}.$$

$$\therefore\ \tan^3\theta-\tfrac{54}{7}\tan^2\theta+\tan\theta+\tfrac{54}{7}=0.$$

From this equation we find $\tan\theta=1.16425$.

$\therefore\ \angle ABC=\angle ACB=81°\ 19'\ 14''$, and $\angle BAC=17°\ 21'\ 32''$.

We now readily find $BC=8.149+$.

II. **Solution by H. C. WHITAKER, C. E., Ph. D., Manual Training School, Philadelphia, Pa.; G.B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; and L. C. WALKER, A. M., Petaluma High School, Petaluma, Cal.**

$$r=(s-a)\tan\tfrac{1}{2}A=\tfrac{1}{2}(b+c-a)\sqrt{\frac{(a+c-b)(a+b-c)}{(a+b+c)(b+c-a)}}.$$

But $b=c$. $\therefore\ r=a\sqrt{\dfrac{2b-a}{2b+a}}$.

$$\therefore\ a^3-2a^2b+4ar^2+8br^2=0.$$

This cubic can be solved, and gives the values of the unknown sides.

Substituting values, we get $a^3-54a^2+49a+2646=0$.

$\therefore\ a=8.1498$, 52.0838, or -6.2336. The first and second values are both admissible. $\cos B=a/2b=.15092$ or $.96451$.

$\therefore\ B=81°\ 19'$, or $15°\ 18'$.

Also solved by *J. SCHEFFER*.

We received a solution of problem 156 from Prof. Henry Heaton too late for credit in the number in which the solution of the problem was published. Professor Heaton gave construction also when the point does not lie between the two parallels. As the construction is similar to the case when the point lies between the two parallels we will not publish the construction.

CALCULUS.

REMARKS ON THE SOLUTION OF PROBLEM 123.

By Wm. E. Heal, Marion, Ind.

A remarkable error occurs in the published solution of Problem 123, in writing $1-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+\ldots=\frac{\pi^3}{35}$.

The summation referred to by Professor Echols and the Editor is incorrectly quoted. The true formula is

$$\tau_{2m+1}=\frac{E_m\pi^{2m+1}}{2^{2(m+1)}(2m)\,!}=1-\frac{1}{3^{2m+1}}+\frac{1}{5^{2m+1}}-\ldots\ldots$$

See Chrystal's *Algebra*, Part II, page 342. For $m=1$ this becomes, since $E_1=1$,

$$\tau_3 = \frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots\cdots$$

I believe the summation of

$$\tau_2 = 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots\cdots$$

is impossible in finite terms. In other words the integral

$$\int_0^{\frac{1}{4}\pi} \log(\tan z) dz$$

is a fundamental form. This integral is given in the great "Tables d'Intégrales Définies" of Bierens de Haan, Table 286, No. 11, in the form

$$\int_0^{\frac{1}{4}\pi} lTgx dx = -\sum_0^\infty \frac{(-n)^n}{(2n+1)^2}.$$

Although the summation in the second member constantly occurs in the above named work, no finite expression is to be found. This is conclusive evidence that no finite expression was known to the author since his results are given in finite terms when possible. For example, in the same table, No. 16, is given

$$\int_0^{\frac{1}{4}\pi} (lTgx)^2 dx = \frac{\pi^3}{16}.$$

Chrystal's *Algebra* gives finite summations for

$$\sigma_{2m} = 1 + \frac{1}{2^{2m}} + \frac{1}{3^{2m}} + \frac{1}{4^{2m}} + \cdots\cdots$$

$$\sigma'_{2m} = 1 + \frac{1}{3^{2m}} + \frac{1}{5^{2m}} + \frac{1}{7^{2m}} + \cdots\cdots$$

$$\sigma''_{2m} = 1 - \frac{1}{2^{2m}} + \frac{1}{3^{2m}} - \frac{1}{4^{2m}} + \cdots\cdots$$

$$\tau_{2m+1} = 1 - \frac{1}{3^{2m+1}} + \frac{1}{5^{2m+1}} - \frac{1}{7^{2m+1}} + \cdots\cdots$$

But neither in that work nor in Jordan's "Traité d'Analyse," nor in the very elaborate "Traité d'Analyse" of Laurent have I been able to find the summation of τ_{2m}. If a finite summation for τ_{2m} exists it seems incredible that it should have escaped the notice of these writers.

REMARK. On again looking up our reference we find that we quoted the reference correctly, though as Mr. Heal observes, it seems that our reference is wrong. The value of the series

$$1-\frac{1}{3^{2n+1}}+\frac{1}{5^{2n+1}}-\frac{1}{7^{2n+1}}+\text{etc.}=\frac{\pi^3}{32}$$

is also given in Carr's *Synapsis of Pure Mathematics*, page 432. Since it now appears that the *value* of the integral is not known in finite terms, we will renew our offer to give the person finding such value a year's subscription to the MONTHLY. ED.

125. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Show that the *complete primitive* of the differential equation

$$\left[\tan^{-1}(x)-\frac{x}{1+x^2}\right]\frac{dy^2}{dx^2}=1\left[\frac{x}{(1+x^2)^2}\right]\left[x\frac{dy}{dx}-y\right]$$

is $y=C\tan^{-1}(x)+cx$.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

w being one solution of

$$\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0\text{.....(1)},$$

and P, Q functions of x, $y=vw$......(2) is given, by the usual theory, from

$$\frac{d^2v}{dx^2}+\left(\frac{2}{w}\frac{dw}{dx}+P\right)\frac{dv}{dx}=0\text{......(3), or } v=c_2+c_1\int\frac{dx}{w^2}e^{-\int Pdx}\text{......(4)}.$$

In the given equation, $w=x$......(5), and

$$P=-\frac{2x^2}{(1+x^2)^2}\div\left[\tan^{-1}x-\frac{x}{1+x^2}\right]\text{......(6)}.$$

(4) now easily becomes

$$v=c_2+c_1\int\frac{dx}{x^2}\left(\tan^{-1}x-\frac{x}{1+x^2}\right)=c_2+c_1\left(\int\frac{\tan^{-1}x}{x^2}-\int\frac{dx}{x(1+x^2)}\right)$$

$$=c_2-c_1\frac{\tan^{-1}x}{x^2}\text{......(7)},$$

and (2) is $y=c_2x-c_1\tan^{-1}x$......(8).